NUMERICAL MODELING OF TURBULENT TRANSFER IN AN EXTERNAL FORCE FIELD

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A mathematical model is used to investigate the specific characteristics of the turbulent structure of unsteady plane-parallel flow of a three-layer fluid.

It is a well-known fact [1, 2] that an external force field can alter the structure of a flow by the generation of spatial nonuniformity and fluctuating motions. In particular, the spatial nonuniformity of the buoyant forces in stratified shear flows results in the generation of vortices, whose axes have a definite orientation relative to the gravitational forces [3]. One of the best-known examples of this situation is the phenomenon of the circulation of mediterranean sea water in the Atlantic Ocean [4]. Although it accounts for only 4% of the waters, it covers an enormous area and retains its own individual character. Such phenomena have been described fairly well in detail and recorded experimentally, e.g., in the investigation of the transfer of passive dyes [5].

An analysis of reported instrumental measurements lends validity to the following assumptions in the derivation of the model equations:

1) The horizontal component of the flow velocity has a mean part and a fluctuation part, while the vertical component has only a fluctuation part.

2) The density of the fluid is a linear function of the "passive impurity" (e.g., salt) concentration.

3) In zones of large density gradients, turbulent transfer is suppressed by gravitational forces [6] and is of the order of the molecular transfer.

We also assume that the pressure fluctuations and the third-order correlation moments are negligible, their contribution being of a diffusive nature and significant only in the event of strong deformations of the mainstream flow [7], which do not take place in our situation. We consider he fluid to be incompressible.

Under the stated assumptions, the Reynolds equations acquire the form

$$\frac{\partial}{\partial t}(\overline{\rho}\overline{u}+\overline{\rho' u'})+\frac{\partial}{\partial z}(\overline{\rho u' w'}+\overline{u}\,\overline{\rho' w'})=-\frac{\partial \overline{P}}{\partial x}+\mu\frac{\partial^2 \overline{u}}{\partial z^2}, \tag{1}$$

$$\frac{\partial}{\partial t}(\overline{\rho' w'})+\frac{\partial}{\partial z}(\overline{\rho u' w'})=-\frac{\partial \overline{P}}{\partial z}-\overline{\rho}g, \tag{2}$$

$$\frac{\partial \overline{\rho}}{\partial t}+\frac{\partial}{\partial z}(\overline{\rho' w'})=D\frac{\partial^2 \overline{\rho}}{\partial z^2}. \tag{3}$$

It must be emphasized that Eqs. (1)-(3) are written in the density fluctuations and not the fluctuations of the impurity concentration, for the simple reason that a linear relationship is assumed between them. We close the system (1)-(3), following Launder [8], by introducing the turbulent momentum-transfer coefficient k_u and the turbulent mass-transfer coefficient k_ρ (for the salt):

$$\overline{\rho' u'}=\overline{\rho' w'}=-k_\rho\frac{\partial \overline{\rho}}{\partial z},\ \overline{u' w'}=-k_u\frac{\partial \overline{u}}{\partial z}. \tag{4}$$

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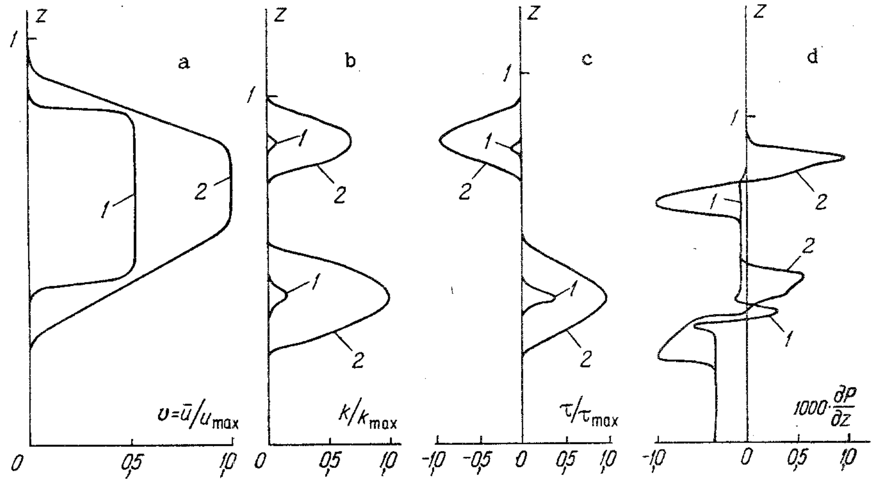

Fig. 1. Distributions of the horizontal velocity (a), turbulent transfer coefficient (b), turbulent frictional stress (c), and vertical pressure gradient (d) according to the numerical model, at two times: 1) t = 3.5; 2) 6.0.

Assuming from now on that the turbulent momentum and mass-transfer processes are similar, we let $k_\rho = \lambda k_u$, $\lambda = \text{const}$, $\lambda < 1$, and for $k = k_u$ we write the transfer equation

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial k}{\partial z} + Ak \frac{\partial \bar{u}}{\partial z} - B \frac{k(k+\nu)}{h^2}, \tag{5}$$

where $\nu = \mu/\rho_0$. The investigated form of the last two terms of Eq. (5), which describe the generation and dissipation k, have been obtained previously [8] by analogy with the expressions for these terms in the turbulent kinetic energy-transfer equation. Replacing the second-order moments according to Eqs. (4), we obtain a closed system of equations, which then forms the basis of our mathematical flow model.

$$\frac{\partial}{\partial t} \overline{\rho u} = \lambda \frac{\partial}{\partial t} k \frac{\partial \bar{\rho}}{\partial z} + \frac{\partial}{\partial z}\left(\bar{\rho} k \frac{\partial \bar{u}}{\partial z} + \lambda \bar{u} k \frac{\partial \bar{\rho}}{\partial z}\right) - \frac{\partial \bar{P}}{\partial x} + \mu \frac{\partial^2 \bar{u}}{\partial z^2}, \tag{6}$$

$$\frac{\partial}{\partial t}\left(\lambda k \frac{\partial \bar{\rho}}{\partial z}\right) - \frac{\partial}{\partial z}\left(\bar{\rho} k \frac{\partial \bar{u}}{\partial z}\right) = \frac{\partial \bar{P}}{\partial z} + \bar{\rho} g, \tag{7}$$

$$\frac{\partial \bar{\rho}}{\partial t} = \frac{\partial}{\partial z}\left(\lambda k \frac{\partial \bar{\rho}}{\partial z}\right) + D \frac{\partial^2 \bar{\rho}}{\partial z^2}. \tag{8}$$

The initial and boundary conditions of the problem are selected as follows: At t = 0 motion is absent, the densities of the layers are $\rho_1 = \rho_0 + \Delta\rho_1$ for the lower heaviest layer, $\rho_2 = \rho_0 + \Delta\rho_2$, $\Delta\rho_1 > \Delta\rho_2$ for the middle layer, and $\rho = \rho_0$ for the top layer. The start of motion of the middle layer is accompanied simultaneously by diffusion of the impurity at its boundaries as a result of the induced constant horizontal pressure gradient. No-slip conditions are assumed to exist at the lower and upper boundaries z = 0 and z = 1.

The difference scheme for the given model was constructed on a hybrid computing grid [9] consisting of nodes Γ_1 for computation of the density field:

$$\Gamma_1 = \{(z_i, t_j),\ z_i = (i + 1/2)\Delta z,\ t_j = j\Delta t,\ \ i = 0, 1, \ldots, 50,\ j = 0, 1, 2, \ldots\}$$

and Γ_2 for computation of the momentum and average-velocity fields, the turbulent mixing coefficient, and the turbulent frictional stress $\tau = -\rho u'w'$;

$$\Gamma_2 = \{(z_i, t_j),\ z_i = i\Delta z,\ t_j = j\Delta t,\ i = 0, 1, \ldots, 50,\ j = 0, 1, 2, \ldots\},$$

where Δz is the spatial step and Δt is the time step of the grid. The values of the density and pressure at the nodes that are not a part of the investigated domain are determined by linear interpolation. The hybrid grid is known [10] to make it possible to avoid damped oscillations in the region of strong gradients and to achieve the density boundary conditions. The usual conditions of the Courant–Friedrichs–Levy type for explicit schemes must be satisfied in order for the resulting system of finite-difference equations to be stable.

The numerical solution of the problem (5)-(8) with the appropriate boundary and initial conditions yields the distributions of $\bar{u}$, k, $\tau = -\overline{\rho u'w'}$, $\partial P/\partial z$ shown in Fig. 1. The computations are carried out for the following values of the parameters: A = 1, B = 1, D = 10^{-8} m^2/sec. g = 9.82 m/sec^2, ρ_0 = 1000 kg/m^3, u_0 = 0.3 m/sec, h_0 = 1 m, μ = 0.002 kg/m·sec.

Figure 1a shows a typical profile of the turbulent flow velocity for a rectangular channel [10] with an abrupt increase (decrease) of the velocity near the intermediate layer with a constant velocity distribution in the upper part of that layer. The plotted distributions of the turbulent transfer coefficient (see Fig. 1b) exhibit a considerable increase in the turbulent transfer in regions with sharp differences of $\partial\bar{u}/\partial z$, i.e., at the interfaces between the fluid layers, and the total absence of turbulent transfer in the middle parts of the layer (where $\partial\bar{u}/\partial z$ is constant). Figure 1c shows the vertical variation of the turbulent frictional stress $-\overline{\rho u'w'}$. As in the case of a rectangular channel [11], $-\overline{\rho u'w'}$ is equal to zero in the middle part of the fluid layers. The maximum (minimum) occurs in the middle layer in the vicinity of the boundary with the heavy (light) fluid and shows that the turbulent friction is the greatest there.

Thus, the nonuniformity of the buoyant forces plays a decisive role in the evolution of turbulence in complex flows. The dynamic pressure field (see Fig. 1d) is restructured in such a way as to ostensibly isolate the moving layer and, as is evident from the figures for the turbulent stress and velocity, the fluid flow behaves like channel flow. Regions with an elevated turbulence level can exist in stratified fluid flows, where they are localized in small volumes by comparison with the scales of the mainstream flow.

NOTATION

$\bar{u}$, average horizontal velocity; u', w', fluctuation parts of horizontal and vertical velocity components; $\bar{\rho}$, ρ', average and fluctuation components of density; ρ_0, characteristic density of water; $\bar{P}$, average pressure; μ, molecular dynamic viscosity coefficient of fluid; D, molecular diffusion coefficient of salt; k_u, turbulent momentum-transfer coefficient; k_ρ, turbulent mass-transfer coefficient of salt; τ, turbulent frictional stress; h, thickness of middle layer; t, time; x, coordinate in the direction of motion; z, coordinate along normal to bottom; A, B, constants.

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